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Kernel-PCA Analysis of Surface Normals for Shape-from-Shading

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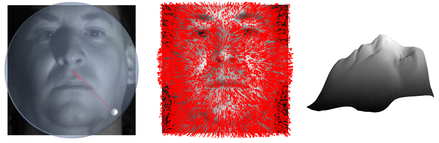
Given the type of object in an image, how can we improve the accuracy of the recovered



- Normals are unit vectors that lie perpendicular to a point on a surface
- Useful in shape recovery

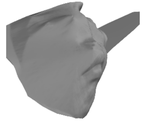
SFS: Recover shape from single image using shading cues
- Given intensities and single light
- Recover parameters that explain intensity values

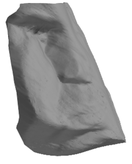
Lambertian reflectance: $E = \rho \mathbf{N} \cdot \mathbf{L}$
- Intensity (E) is a function of the normals ($\mathbf{N}$), light vector ($\mathbf{L}$) and the albedo (ρ)

- Under-constrained problem - **infinite number of solutions!**
- **Constrain the problem by introducing priors on the normals!**

State-Of-The-Art

- Large literature, but most techniques struggle with complex objects such as faces
- Barron & Malik [1] is the state-of-the art general SFS algorithm
  - Introduce a general set of priors
  - Recovers accurate lighting but *not shape*

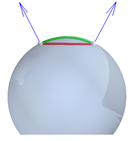




Proposal

Build a statistical model of normals by performing component analysis

- Performing component analysis on normals is non-trivial
- Normals can be seen as embedded on the 2-sphere



- Green arc represent the correct distance between the two normals, a geodesic on the sphere.
- Red line indicates the distance calculated by euclidean Principal Component Analysis (PCA).

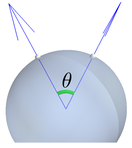
$$\phi : \mathbb{R}^F \rightarrow \mathcal{H}, \quad \mathbf{x} \rightarrow \phi(\mathbf{x}) \qquad \langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle = k(\mathbf{x}_i, \mathbf{x}_j)$$

Kernel-PCA: Calculate non-linear distances between normals by defining kernels that accurately measure similarities.

Propose two novel kernels

Inner Product (IP)

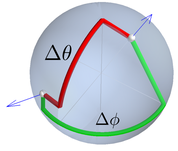
$$k(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{N} {\mathbf{n}_k^i}^T \mathbf{n}_k^j = \sum_{k=1}^{N} \cos \theta_k^{ij}$$



Equivalent to standard PCA *without* mean subtraction

Spherical (SPHER)

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{N} \cos(\Delta\phi_k^{ij}) + \sum_{k=1}^{N} \cos(\Delta\theta_k^{ij})$$

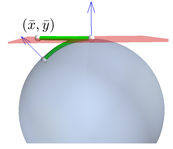


Simple vectorized form

Embed two existing projections

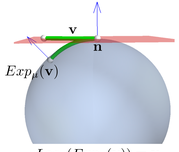
Azimuthal Equidistant Projection (AEP)

$$k(\mathbf{x}_i, \mathbf{x}_j) = \left[\bar{x}_k^1, \bar{y}_k^1, \ldots, \bar{x}_k^N, \bar{y}_k^N\right]^T$$



Principal Geodesic Analysis (PGA)

$$k(\mathbf{x}_i, \mathbf{x}_j) = \left[Log_{\mu^1}(\mathbf{n}_k^1), \ldots, Log_{\mu^N}(\mathbf{n}_k^N)\right]^T$$



$Log_\mu(Exp_\mu(\mathbf{v})) = \mathbf{v}$

Results

Embedded the kernel framework into an existing SFS algorithm [2]

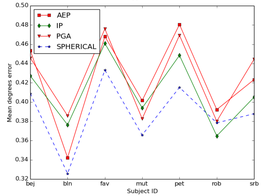
Geometric SFS [2] Algorithm

Iterate until $\sum_{i,j} \arccos\left(\mathbf{n}(i,j)' \cdot \mathbf{n}(i,j)''\right) < \epsilon$:

1. Calculate an initial estimate of the surface normals.
2. Project the needle-map into the feature space using one of the projection operators: $\phi_F(\mathbf{x})$
3. Reconstruct the best fit feature space vector: $\mathbf{U}_F {\mathbf{U}_F}^T \phi_F(\mathbf{x})$.
4. Use the inverse mapping to recreate a set of surface normals, $x' = {\phi_F}^{-1}\left(\mathbf{U}_F {\mathbf{U}_F}^T \phi_F(\mathbf{x})\right)$, with individual normals $\mathbf{n}(i,j)'$
5. Enforce hard-irradiance constraint on the reconstructed normals to find the on-cone surface normal, $\mathbf{n}(i,j)''$

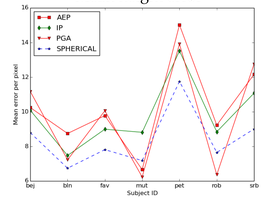
Celebrity Images

Mean angular error



Photoface Database

Mean height error



References

[1] J. Barron and J. Malik. Shape, illumination, and reflectance from shading. Technical report, EECS, UC Berkeley, May 2013.

[2] W. A. Smith and E. R. Hancock. Recovering facial shape using a statistical model of surface normal direction. IEEE T-PAMI, 28(12):1914-1930, 2006.